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LLNL-JRNL-407048

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September 16, 2008

Physical Review Letters

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Abstract

We propose to exploit a self-focusing effect in the atmosphere to assist delivering powerful laser beams from orbit to the ground. We demonstrate through numerical modelling that if the self-focusing length is comparable with the atmosphere height the self-focusing can reduce the spot size on the ground well below the diffraction limited one without beam quality degradation. The use of light self-focusing in the atmosphere can greatly relax the requirements for the orbital optics and ground receivers.

The abundance of solar energy outside the Earth makes attractive the electricity production in space. Harnessing and accumulation of the solar energy at the orbit stations for further wire less power transportation to the Earth is one of the global concepts of renewable energy sources [1,2]. The accumulated solar power can be used to produce focussed electromagnetic beams of either microwave waves or laser radiation. Despite many attractive features and the first scientific developments, [1-3] this technology is far from being practical with many technical and conceptual issues yet to be resolved before it becomes a real competitor to the existing power technologies. Initial proposals have been based on the energy transportation by microwaves. However, recent progress in laser science has stimulated research into the feasibility of using laser-based orbit systems in which laser radiation is utilized for transport of converted solar energy to the ground. The use of lasers instead of microwaves would allow greatly reduced transmitter and receiver sizes. In this Letter we focus on the one of the key problems of reducing the size of the transmission and receiving facilities. Indeed, even for a diffraction limited beam, a large precise focusing optics in the space and large receiving facility on the ground would be required. We propose a general approach that reduces these size requirements by exploiting a self-focusing effect to deliver powerful laser beams to the ground. The idea to use self-focused beams for energy transportation was suggested just after discovery steady state channels where self-focusing compensated diffraction [4]. Soon after it was shown that steady state propagation is always unstable [5]. For beams with power over a threshold critical power, filamentation, collapse and uncontrolled ionization result in beams break. However, the situation can be different for propagation from orbit through the inhomogeneous Earth's atmosphere. We demonstrate that when the self-focusing length is comparable with the atmosphere height, the catastrophic self-focusing can be greatly suppressed and a smooth compression of the whole beam is possible. Numerical modelling demonstrates that the controlled beam compression is well below the diffraction limited spot size permitting size reductions in receivers on the ground and focusing optics in the space.

To illustrate the idea without loss of generality, consider vertical laser beam propagation. The full analysis based on using the detailed model taking into account all important physical effects having impact on beam propagation will be presented elsewhere, but here we focus only on the key physical effects that are relevant to illustration of the proposed concept. The key features of the beam evolution (diffraction and Kerr nonlinearity) can be described using the standard paraxial approximation for the envelope of the electric field - the Nonlinear Schrödinger equation (NLSE):

$$i\frac{\partial A}{\partial z}+\frac{1}{2n_0k_0}\Delta_\perp A+n_2k_0\,|A|^2\,A=0 \qquad (1)$$

Here z is the direction of propagation, with $z=0$ the sea level, and the propagation from the orbit to the ground. The initial condition for this Cauchy problem, beam profile and phase waveform at the height of the orbit, $z=F$ **is assumed to be,**

$$A(z=F,r)=\sqrt{\frac{P}{\pi R_0^2}}\exp[-\frac{(1+iC_0)}{2R_0^2}r^2].$$

Here $D=2R_0$ is the mirror diameter, $C_0=k_0R_0^2/F$, and F is the focal distance or height of the orbit. In the limit of strong pre-focusing ($C_0>>1$), when the focal spot is much smaller then the focusing mirror the beam has a Gaussian non-focused shape on the ground with the radius

$$R_{\min}=R_{ground}=\frac{\lambda_0 F}{\pi D} \qquad (2)$$

Typically, when the Rayleigh length is comparable to or longer than the atmosphere height, the beam size entering atmosphere R_{atm} is close to it footprint on the ground $R_{atm}\approx R_{ground}$. Expression (2) relates the focusing mirror size laser wavelength, laser footprint on the ground R_{ground} and the orbit height F. It is seen from Eq. (2) that the beam spot size at the ground is inversely proportional to the orbital mirror diameter imposing some requirements on the orbital mirror size. Evidently, technical design issues depend on many factors and it is likely that different solutions can be proposed. Here we would like to stress an additional degree of freedom that appears when considering nonlinear effects during beam propagation from the orbit to ground in the atmosphere. Namely, under certain conditions a nonlinear lens effect provided by the self-focusing might reduce the beam footprint on the ground allowing for a larger initial size of a beam entering atmosphere R_{atm}, and consequently smaller and lighter transmitter mirrors could be used on orbit. In this Letter we present basic analysis of the impact of the self-focusing on transportation of laser beam from an orbit to the ground.

It should be noted that the textbook formula (2) is valid only when the focal spot $R_{\min}$ is much smaller the mirror radius R_0. In our case of long distance focusing, $R_{\min}$ can be comparable with R_0 and this must be taken into account. In the general case, the minimal beam waist is modified to be,

$$R'_{\min} = \frac{R_{\min}}{\sqrt{1+\left(\frac{R_{\min}}{R_0}\right)^2}} = \frac{\lambda_0 F/(\pi D)}{\sqrt{1+\left(\frac{\lambda_0 F/\pi D}{R_0}\right)^2}} \qquad (3)$$

and the location of minimum (waist) is shifted toward the mirror with a new focus at

$$F' = \frac{F}{1+\left(\frac{R_{\min}}{R_0}\right)^2} = \frac{F}{1+\left(\frac{\lambda_0 F/\pi D}{R_0}\right)^2} \qquad (4)$$

It is easily seen that in the linear theory (3)-(4) under conditions of maximal compression at the ground (or in other words, assuming that the distance from a laser at the orbit down to receiver L is equal exactly to F'), the radius of the mirror R_0 cannot be less than a certain minimal value given by $R_0 \geq R_0^{(1)} = \sqrt{L\lambda_0/\pi}$. The minimum possible mirror radius corresponds to $F = 2L$ (and $C_0 = 1$). In physical terms, this means that at small mirror size the beam can not be focused at all, because F in this case becomes comparable with the Rayleigh length. Note also that a linear compression factor of the laser beam with such a "minimal-diameter orbital mirror" is limited by square root of 2: $R'^{(1)}_{\min} = R_0^{(1)}/\sqrt{2} = \sqrt{L\lambda_0/(2\pi)}$.

The nonlinearity in (1) is a function of altitude. The nonlinear refractive index is proportional to density. The density can be interpolated as exponential (isothermal atmosphere) and the height scale parameter $h \approx 6\ km$. Introducing the density at sea level ρ_0, the corresponding nonlinear refractive index as a function of z is,

$$n_2(z) = n_2(0)\frac{\rho}{\rho_0};\ \rho = \rho_0 e^{-\frac{z}{h}}$$

Here the refractive index on the ground is $n_2(0) = 5.6\times10^{-19}\ cm^2/W$. Note that at large distances from the ground the contribution the nonlinear term in (1) is negligible, but it becomes more and more important when the beams approach sea level.

Self-focusing in homogeneous media starts when the beam power exceeds the critical value, $P_{cr} = 11.68\lambda_0^2/(8\pi^2 n_0 n_2) = 0.93\lambda_0^2/(2\pi n_0 n_2)$. At $z = h = 6\ km$, $P_{cr} = 4.6\,GW$ for $0.8\ \mu m$ light. The high value for this critical power means that short pulse lasers must be used for energy transportation schemes making use of the self-focusing effect. In homogeneous media the beam experiences some compression even at power levels below P_{cr}, above approximately $P \approx 0.7\,P_{cr}$. Gaussian beams compress as a whole up to a radius of ~ 0.5 of the diffraction limited one. At higher powers the central filament becomes important, forming a singularity when P approaches the critical value [6]. When P is above the critical value, the beam collapses to a singularity at a distance L, which for the unfocused beam with radius a is given by, $L \sim \dfrac{ka^2}{\sqrt{P/P_{cr}-1}}$. For powers well above P_{cr}, the beam breaks up in multiple filaments, each filament having P~P_{cr} and collapsing independently of the other filaments. When the sample size (propagation distance in the medium) is smaller than L, the flat top over time beam can also compress up to a size much smaller than the diffraction limited spot size, however, the compression scenario is very sensitive to variations of the initial parameters. In practical terms, compression up to 0.5 of the diffraction limited spot at $P \le 0.7\,P_{cr}$ is possible without strict restriction on beam parameters stability [7].

In what follows it is convenient to use as a reference the P_{cr} at $z = h$. For P well below P_{cr}, the self-focusing length exceeds the atmosphere thickness and the propagation is linear. When L is much smaller than h, the self-focusing of beam takes place in what is an essentially a homogeneous media. For a beam diameter of $\sim 10\,cm$ and $P \cong P_{cr}$, the self-focusing length is about the scale of the density variation of the atmosphere. In this case one can expect that the density variation will suppress the beam filamentation and provides the self-focusing of the beam as a whole. Below we present the results of numerical modelling (1) supporting these arguments.

Consider energy transport from a low earth orbit in which a Gaussian beam is focused by a mirror and propagated to the ground from a height of $500\,km$ in all modeling below. The two key parameters we vary in our simulations are the mirror diameter and the beam power. It was demonstrated in early studies of self-focusing that the filamentation threshold in the axi- symmetric case (formation of the ring structures) takes place at approximately the same place as filamentation within the complete model of (1). Therefore, without loss of generality, we consider the axi-symmetric problem to prove the concept. For $P \sim P_{cr}$ the variation of atmospheric refractive index starts to affect the beam propagation below approximately $20\,km$. Our modelling results clearly demonstrate that in inhomogeneous medium up to some power level the compression of the whole beam is possible without splitting up into multiple sub beams. Figure 1 shows the intensity distribution on the ground for a mirror radius $R_0 = 1\,m$ and for several beam powers. One can observe a strong (factor of five) beam compression in comparison with linear propagation without any indication of filamentation.

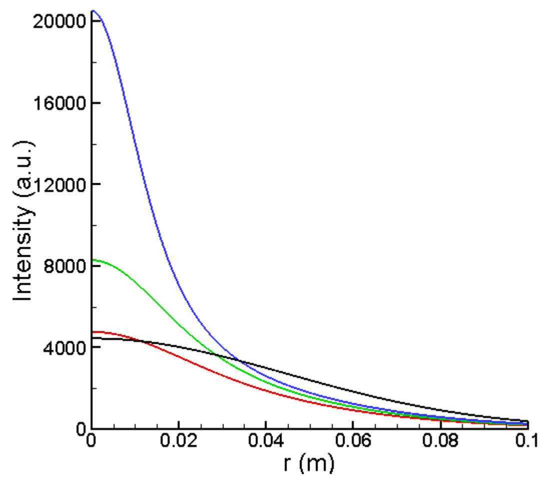

Fig. 1. Beam intensity distribution on the ground for R_0 =1m: the red line - $P/P_{cr} = 1.0$, the green line - $P/P_{cr} = 1.29$, the blue line - $P/P_{cr} = 1.66$ The black line corresponds to the linear diffraction.

The compressed beam radius $<r> = \dfrac{\int_0^\infty rI(r)rdr}{\int_0^\infty I(r)rdr}$ at the ground as a function of initial power for different mirror radii is presented in Fig. 2. It is seen that for $R_0 = 1\,m$ mirror up to $P/P_{cr} \approx 2$ the compression is not very sensitive to power variations. The beam experiences self-focusing with an average radial compression of 13.95 from 10.6 cm to $0.76\,cm$ at $P/P_{cr} = 2$. Further power increases result in a beam filamentation and splitting near the ground. We note that the results of calculations shown here are robust, being non-sensitive to the numerical noise and small variations of initial conditions. For the case of $R_0 = 0.5\,m$ filamentation can be suppressed for powers up to $P/P_{cr} \approx 12$. For $R_0 = 0.357$m the radius at the ground versus beam power dependence looks similar, but the power required for compression is much higher. For $R_0 = 0.357$m filamentation starts with powers as high as $\dfrac{P}{P_{cr}} = 134.6$. Note that at $P/P_{cr} \approx 2$ the value of a beam power at the ground measured in units of critical power at the ground is $P/P_{cr}(0) \approx 5.5$ for $R_0 = 1\,m$ indicating the self-focusing suppression by density inhomogeneity, the central result of this investigation. The effect is even more pronounced for $R_0 = 0.357$m where the beam power at the ground measured in units of critical power at the ground is as high as. $\dfrac{P}{P_{cr}(0)} = 365.9$

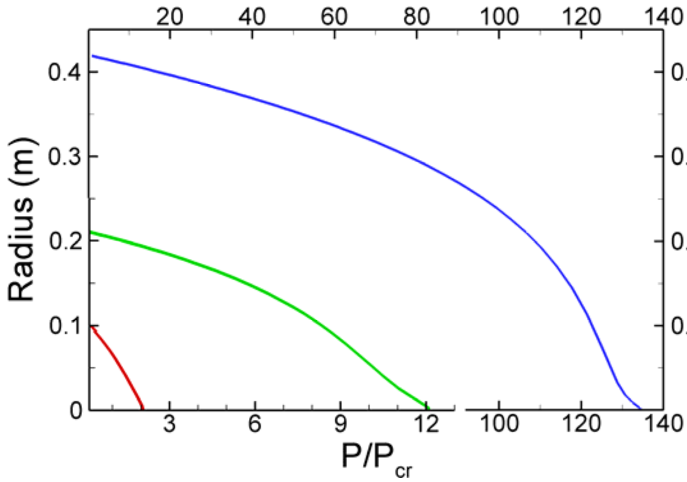

Fig. 2. The beam radius at the ground versus power for the mirror radius $R_0 = 1\,m$ (red line) and $R_0 = 0.5\,m$ (green line) and $R_0 = 0.357\,m, C_0 = 1$ (blue line)

Qualitatively the suppression of self-focusing in atmosphere can be understood by analysis of the modulation instability of plane wave with amplitude A. The maximum growth rate due to modulation instability corresponds to the transversal perturbations with a scale $l^2 \sim 1/k^2 n_2 A^2$ [8]. During beam compression in a homogenous medium, the intensity increases and the beam size decreases in such a way that the beam size is always near the most unstable transversal scale corresponding to the beam intensity. In the considered inhomogeneous problem, due to effective increase of the nonlinear refractive index, the most unstable scale becomes smaller than the beam size and breaks the matching condition between the beam intensity, beam radius and the most unstable scale point, ie, self-focusing development is slowed down. In other words, during the propagation in a homogenous medium the power of the beam becomes higher than the critical power and it must break into filaments. However, in practical bell-shaped beams the initial level of the most unstable short-wave perturbations is low and it takes additional distance for them to grow up. From the view point of propagation along the finite length medium with an observer at the output edge this effectively looks as a suppression of the filaments growth.

We note an interesting analogy between propagation of laser beam in the atmosphere and in propagation in an amplifying medium. By applying the substitution $A \to Ae^{z/2h}$ the problem under consideration that is governed by the Eq. (1) can be transformed into the problem of beam evolution in a uniform amplifying media with an effective gain $g = 1/2h$. Therefore, some features of the light propagation from the orbit can be modelled by experiments with a laser amplifier and one can expect the self-focusing suppression when the product of the gain to self-focusing length $g\,L$ is of the order of unity. In a similar way the beam propagation up to space can be model by the propagation through the media with corresponding attenuation.

[8]

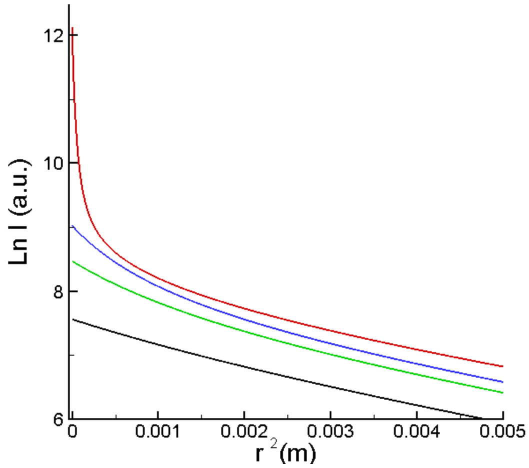

Fig. 3. Logarithm of the field intensity $\ln I$ versus r^2. The black line - $P/P_{cr} = 0.55$, the green line - $P/P_{cr} = 1.0$, the blue line - $P/P_{cr} = 1.29$, the red line - $P/P_{cr} = 2.0$, $R_0 = 1\,m$

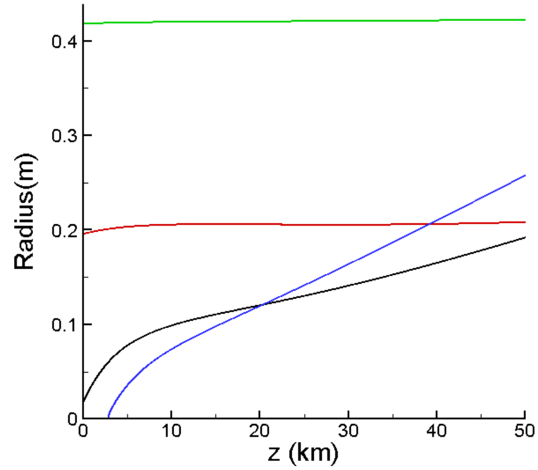

Fig. 4. The beam radius versus distance z for mirror radius $R_0 = 1.5\,m$ - blue line, $R_0 = 1\,m$ - black line and $R_0 = 0.5\,m$ – red line, $R_0 = 0.357\,m, C_0 = 1$ - green line; $P/P_{cr} = 1.84$.

An important characteristic of the received laser beam is its spatial distribution. Figure 3 shows a beam structure near the ground in a logarithmic plot of the field intensity as a function of the square of a transversal coordinate radius - so that a Gaussian beam is presented here by a descending straight line. It is seen that up to some power level a laser beam is focused preserving its Gaussian shape. However, at higher powers the distribution acquires two distinctive scales: the central peak and the Gaussian background, with the central core containing most of the beam energy. Evolution of the spike intensity near the ground follows scaling $\propto 1/(z - z_*)$, consistent with singularity scaling in self-focusing theory. Beam compression can be controlled by suitable arrangement of the pre-focused phase waveform and beam power. Figure 4 presents results of modelling of the beam average radius evolution in the atmosphere for different mirror radii. For larger mirror radii the size of the beam entering the atmosphere is smaller. As the self-focusing length is approximately proportional to the square of the input beam radius, for smaller spot size beams the self-focusing length is short and the beam might collapse before it reaches the ground. Of course, physical singularity formation is stopped by higher order effects not included in Eq. (1). For the small mirrors the beam size, entering atmosphere is large and the characteristic self-focusing length might longer than the atmospheric propagation distance and so higher power is required to compress the beam on the ground. In this case one can observe noticeable beam compression without loss of beam quality

Consider now the problem of energy transportation from orbit. For a fixed orbit height F the system design parameters are the beam power and focusing mirror diameter D. As it was mentioned above, for the typical situations the Rayleigh length is shorter than or comparable to the atmospheric height and the beam entering atmosphere can be treated as a unfocused Gaussian beam with size given by Eq. (2). Now we can relate the self-focusing length L, the orbit height and the mirror diameter to the input beam power corresponding to the maximal compression without filamentation. The condition $L \cong h$ leads to the following relation:

$$\frac{P}{P_{cr}} - 1 \sim \left(\frac{\lambda F^2}{hD^2} \right)^2 \tag{5}$$

Thus, the power is related to the mirror diameter as,

$$D^4 \left(\frac{P}{P_{cr}} - 1 \right) = const \tag{6}$$

Equation (6) presents an effective design rule for maximal compression without filamentation of the transported beam. The results of the numerical modelling presented in Fig. 5 confirm that apart from the small region corresponding to powers close to P_{cr}. Eq. (6) holds and can be used in estimate of the beam/system parameters that allow for laser power transportation without beam splitting and filamentation. Two effects limit the applicability of the scaling relation (5). For large mirror radii, the Raleigh length becomes comparable with atmosphere height (for R>10 cm), so the estimates for beam parameters entering the atmosphere must be modified. For small mirror sizes, the footprint on the ground can be comparable with mirror radius and beam parameters must be modified according (3)-(4).

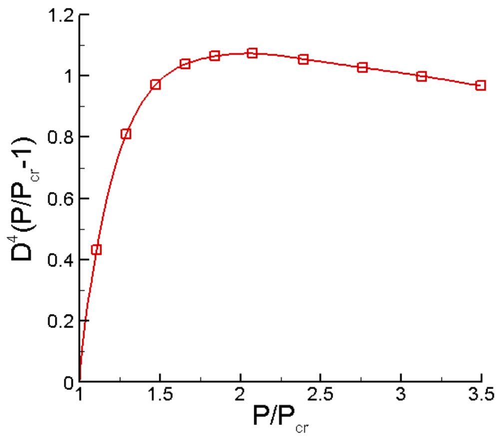

Fig. 5. The dependence of $D^4\left(P/P_{cr}-1\right)$ on power required to start the filamentation P/P_{cr}

Space to ground power beaming system design can be simplified using a desirable beam compression rate. We have derived a general scaling relation useful for this task relating the mirror diameter to the input beam power corresponding to the maximal compression without filamentation Eq. (6). It should be stressed that for high compression regimes the focusing becomes very sensitive to the initial beam parameters, a situation that is not practical from an engineering view point. For example, it is difficult to control pulse energy variations. Even more important is that in such high power regimes, scattering by atmospheric turbulence, beam aberrations and air breakdown are likely to be much more important than in low power situations. We would like to stress that in this Letter we present a concept rather than a thorough examination of all details describing powerful laser beam propagation from orbit to ground through the inhomogeneous terrestrial atmosphere. Detailed analysis will be presented elsewhere. The key result presented here is that that the beam footprints on the ground can be made small without running into filamentation and nonlinear beam break-up. This expands the possibilities for power collection on the ground. As an example, one can consider the concept of a power sphere [9]; an integrating sphere to efficiently captures and converts the laser beam to electricity. Note also that the proposed approach not only allows a reduction in footprint on the ground, but also helps to scale down the required space mirror size, reducing the weight and cost to deploy these systems in space.

Conclusion

We have demonstrated that the self-focusing in atmosphere can substantially compress the laser beam without beam quality degradation. The atmosphere density variation suppress the filamentation process and makes possible the nonlinear lens-assisted delivery of the high quality laser beam to a small spot size on the ground.

This work was partially performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344 and interdisciplinary grant 31 from the Siberian Branch of the Russian Academy of Science. The financial support of the British Council and the Royal Society is acknowledged.

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